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**EVALUATION OF ANALYTICAL MODELS FOR PREDICTION OF HEIGHT OF
CAPILLARY RISE FROM WATER TABLE IN DIFFERENT POROUS MEDIA**

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ABSTRACT

The prediction of the capillary rise height in the unsaturated porous media above the water table is important in hydrology, water management, contaminant transport and geotechnical engineering studies. The heights of the capillary rise were predicted by the two analytical models; one based on the exponential function of the hydraulic conductivity proposed by Gardner alongwith the use of its parameter equivalence with Mualem-based van Genuchten hydraulic conductivity function, and the other based on the power-law conductivity function of Brooks-Corey derived with the use of corresponding soil water retention function in the Burdine model of relative hydraulic conductivity. The parameters of these models were independently evaluated from the corresponding soil water retention functions using the non-linear least squares optimization technique. The predicted heights of the capillary rise were compared with the observed capillary rise height and it was observed that the predicted heights of the capillary rise were found to be more closer to the observed capillary rise height in the medium-textured porous media of sandy loam and loam soils and moderately fine-textured silty clay loam soil. More deviations between the observed and predicted capillary rise heights were observed in the coarse-textured soil.

KEYWORDS: Porous media, capillary rise models, performance evaluation.

INTRODUCTION

The capillary rise is a well-known phenomenon in the unsaturated porous media that describes the upward movement of the pore water from the water table and is driven by the hydraulic gradient and the hydraulic conductivity depending upon the texture of the porous media i.e. soil. Lu and Likos [1] reported that the fundamental characteristics related to the capillary rise i.e. the height, water storage capacity and the rate of capillary rise are of practical importance. The study of capillary rise is helpful in the implementation of unsaturated soil mechanics into the geotechnical engineering practices, in analyzing the process of soil salinization especially under shallow brackish water table conditions, to meet some of the water requirements of the crops in irrigated as well as in unirrigated agriculture and analyzing the contaminant transport process in implementing the environmental engineering practices.

Various investigators have developed analytical models of the capillary rise dynamics beginning with Terzaghi [2] and followed by Gardner [3], Anat et al. [4], Anat and Sukghaem [5], Cisler [6], Eagleson [7], Warrick [8], Malik et al. [9], Parlange et al. [10], Salvucci [11], Lu and Likos [1], Aghajani et al. [12] and Sadeghi et al. [13]. Too et al. [14] reported that most of these models either used soil water suction head-based exponential or power-law hydraulic conductivity functions of various types proposed by different investigators and embedded these functions in the Darcy's law to develop analytical models of the capillary rise. The parameters of these capillary rise models were either estimated by fitting empirical relations with the soil hydraulic and/or physical properties or by using corresponding soil water retention functions.

From the perusal of the analytical models of capillary rise of Gardner [3], Anat et al. [4] and Cisler [6] it is evident that the capillary water flux is inversely proportional to the height of the capillary rise from the water table with a power of some exponent and with different values of the constant of proportionality. The Gardner [3] model is a special case of the Cisler model [6] which used power-law unsaturated hydraulic conductivity function. From the

perusal of the analytical models developed by Cisler [6] and Warrick [8], it is seen that the Cisler [6] model for the steady state capillary rise flux as a function of capillary rise height from the water table is a special case of Warrick model [8]. Malik et al. [9] used exponential unsaturated hydraulic conductivity function in the Darcy's law and showed analytically that after some initial time had elapsed, capillary water flux is inversely proportional to the exponential of the height of the capillary rise from the water table with some constant of proportionality. The parameters in this model were empirically evaluated as fitted parameters in terms of the soil hydraulic properties of saturated hydraulic conductivity and the wilting point. Parlange et al. [10] suggested the improvement in the model of Malik et al. [9] based on Haverkamp et al. [15] in which the height of the capillary rise in the capillary fringe was analyzed in terms of a parameter related to the soil water suction head at air-entry while the height of the capillary rise above the capillary fringe was shown as proportional to the logarithm of a variable relating to the inverse of capillary water flux with some constant of proportionality defined in terms of some weighted integral of the hydraulic conductivity. They suggested that these parameters i.e. one related to the air-entry suction head and the other related to the weighted integral of the hydraulic conductivity can be estimated as fitted parameters from the experimental data of the height of the capillary rise above the water table and the corresponding capillary water flux for different porous media with the known values of saturated hydraulic conductivity but the model proposed by Parlange et al. [10] was not evaluated. As is seen from the literature, the prediction behavior of these models is affected by the types of hydraulic functions used and the method for evaluating the parameters of these models was used.

In this study, the heights of the capillary rise were predicted using the analytical models based on the exponential as well as the power-law hydraulic conductivity functions and using the independently evaluated parameters of these models from the soil water retention functions and the corresponding theoretical hydraulic conductivity functions and were compared with observed capillary rise data.

CAPILLARY RISE MODELS

One-dimensional unsteady water flow in vertical direction from the fixed water table in the unsaturated, homogenous and isotropic porous media of soil is described as:

$$\frac{\partial \theta}{\partial t} = - \frac{\partial q}{\partial z} \quad (1)$$

Where θ is the volumetric soil water content [L^3/L^3], z is the vertical distance [L] measured positive upward from the fixed water table with $z = 0$ at the water table, t is the time [T] and q is the upward capillary water flux [L/T] entering the porous media at $z = 0$ from the fixed water table. As stated the capillary water flux in reality is unsteady as described by Eq. (1) but for the infinitesimal small time intervals, the capillary water flux can be assumed steady and under this assumption the capillary water flux is described by the Darcy's law as:

$$q = - K(h) \frac{\partial H}{\partial z} \quad (2)$$

Where H is the total hydraulic head [L] represented by the sum of the soil water suction head h [L] and the gravitational head z [L] neglecting the osmotic head. $\partial H / \partial z$ is the total hydraulic gradient causing the upward movement of soil water from the fixed water table and $K(h)$ is the unsaturated hydraulic conductivity [L/T] as a function of soil water suction head h [L]. The suction head h at the water table is taken as zero. From Eq. (2) the, capillary rise profile as a function of soil water suction head in the unsaturated infinitesimal small volume of the soil from the fixed water table is described as:

$$dz = \frac{dh}{1 + [q/K(h)]} \quad (3)$$

Haverkamp et al. [15] emphasized that the water flow in the soil with almost at saturation condition (soil water suction head h becomes almost constant) needs to be analyzed separately and based on this condition as is available in the capillary fringe in case of the capillary rise Parlange et al. [10] dealt the capillary rise in the capillary fringe and above it separately. They replaced $K(h)$ with K_s (hydraulic conductivity at saturated soil water content θ_s) for the soil water suction head condition $0 \leq h \leq h_a$; 0 and h_a are the suction heads at the water table and at the maximum height of the capillary fringe, respectively and h_a is taken as the air-entry suction head which is the minimum suction head required to drain the largest pore in the soil, Thus the total height of the capillary rise above the water table ($z = 0$) to the ultimate wetting front position ($z = Z$) of the capillary rise is written as:

$$Z = \int_0^{h_a} \frac{dh}{1+q/K_s} + \int_{h_a}^{h_2} \frac{dh}{1+q/K(h)} \quad (4)$$

Where h_2 is the soil water suction head at the ultimate wetting front position of the capillary rise. To arrive at an analytical solution for the capillary rise, the appropriate form of unsaturated hydraulic conductivity function $K(h)$ needs to be incorporated in Eq. (4). Leij et al. [16] reported that different types of unsaturated hydraulic conductivity functions either empirical or theoretical in nature have been developed by various investigators. Some of these functions can be integrated analytically while others can be integrated numerically and use of a specific form of a function affects the capillary rise behavior.

For developing the analytical model of the capillary rise, firstly the exponential conductivity function proposed by Gardner [3] and as modified by Philip [17] by including the finite air-entry soil water suction head h_a was used and is described as:

$$K(h) = K_s e^{-\alpha_G(h-h_a)} \quad (5)$$

Where $\alpha_G [L^{-1}]$ is a parameter linked to the measure of the pore size distribution in the soil and represents the rate of reduction in the hydraulic conductivity with the increase in the soil water suction head h . Embedding Eq. (5) in Eq. (4) yields as:

$$Z = \frac{h_a}{1+(q/K_s)} + \int_{h_a}^{h_2} \frac{dh}{1+(q/K_s) e^{\alpha_G(h-h_a)}} \quad (6)$$

Substituting $t = 1 + (q/K_s) e^{\alpha_G(h-h_a)}$ in Eq. (6) with limits $t_1 = 1 + (q/K_s)$ and $t_2 = 1 + (q/K_s) e^{-\alpha_G h_a} e^{\alpha_G h_2}$ and on rearranging Eq. (6) yields as:

$$Z = \frac{h_a}{1+(q/K_s)} + \frac{1}{\alpha_G} \int_{t_1}^{t_2} \left[\frac{1}{t-1} - \frac{1}{t} \right] dt \quad (7)$$

Integrating and on rearranging Eq. (7) yields as:

$$Z = \frac{h_a}{1+(q/K_s)} + \frac{1}{\alpha_G} \left[\ln \left\{ \frac{1-(1/t_2)}{1-(1/t_1)} \right\} \right] \quad (8)$$

Substituting the values of the limits $[t_1, t_2]$ in Eq.(8) and on rearranging yields as:

$$Z = \frac{h_a}{1+(q/K_s)} + \frac{1}{\alpha_G} \left[\ln \left(\frac{q e^{\alpha_G h_2}}{K_s e^{\alpha_G h_a} + q e^{\alpha_G h_2}} \right) - \ln \left(\frac{q e^{\alpha_G h_a}}{K_s e^{\alpha_G h_a} + q e^{\alpha_G h_a}} \right) \right] \quad (9)$$

On further rearranging, Eq. (9) is expressed as:

$$Z = \frac{h_a}{1+(q/K_s)} + \frac{1}{\alpha_G} \left[\ln \left\{ \frac{q+K_s}{q+K_s e^{-\alpha_G(h_2-h_a)}} \right\} \right] \quad (10)$$

Eq. (10) is a generalized form of the capillary rise for the finite value of the soil water suction head $h = h_2$ at the ultimate height of the wetting front of the capillary rise. For a limiting case, the soil water suction head at the ultimate height of the wetting front of the capillary rise h_2 is assumed to be infinity and under this assumption Eq. (10) yields as:

$$Z = \frac{h_a}{1+(q/K_s)} + \frac{1}{\alpha_G} \left[\ln \left\{ 1 + \frac{K_s}{q} \right\} \right] \quad (11)$$

The analytical expression (Eq. 11) also proposed by Parlange et al. [10] estimates the height of the capillary rise above the water table in different soils knowing the corresponding capillary water flux at that height and the corresponding values of h_a , α_G and K_s of these soils. For estimation of h_a and α_G parameters of different soils, indirect method using the soil water retention functions developed from the easily measured soil water retention data was used. For

estimating these parameters, no soil water retention function was proposed by Gardner for the corresponding unsaturated hydraulic conductivity function as used in this analysis though Russo [18] formulated soil water retention function to arrive at the Gardner exponential conductivity function by incorporating the water retention function in Mualem model [19] of relative hydraulic conductivity. Too et al. [14] observed that the five-parameter function of van Genuchten [20] performed better than the four-parameter function of Russo [18]. So in this analysis, the equivalence between the parameters of the hydraulic conductivity functions of Gardner [3] and van Genuchten [20] was considered.

Various investigators namely Birkholzer et al. [21] Furman and Warrick [22], Morel-Seytoux et al. [23], Rucker et al. [24] and Ghezzehei et al. [25] proposed parameter equivalence between the Gardner and van Genuchten functions. Based on the agreements between these two functions, the parameter equivalence between the hydraulic conductivity functions of Gardner [3] and Mualem [19]-based van Genuchten [20] proposed by Ghezzehei et al. [25] was used in this study. They reported that the Gardner parameter α_G is linearly related with the van Genuchten parameters of n and α_{VG} as:

$$\alpha_G \approx 1.3 n \alpha_{VG} \quad (12)$$

The parameter n is dimensionless and is related to the spread of the pore-size distribution in the porous media and the parameter α_{VG} [L^{-1}] is a constant related to the modal pore-size. However, Ghezzehei et al. [25] reported that the conversion of h_a in terms of van Genuchten parameters results in unphysical and unrealistic air-entry value for $n \leq 2$ i.e. it fails to assess the h_a realistically for the whole range of n . So in this analysis for estimating the value of h_a , the power-law function of soil water retention of Brooks-Corey [26] which introduced a well-defined air-entry value was used as:

$$\theta = \theta_r + (\theta_s - \theta_r) (\alpha_{BC} h)^{-\lambda} \quad (13)$$

Where θ [L^3/L^3] is the volumetric soil water content at the soil water suction head h . θ_r and θ_s are the residual and saturated volumetric soil water contents, respectively. The parameter λ ($\lambda > 0$) is the pore-size distribution index affecting the slope of the soil water retention function and characterizes the width of the pore-size distribution of the soil medium. The parameter α_{BC} [L^{-1}] representing the desaturation rate of soil water is related to the pore-size distribution and whose inverse is referred to the soil water suction head at air-entry (h_a) as:

$$h_a = 1/\alpha_{BC} \quad (14)$$

The parameters α_G and h_a in Eqs. (11) were translated in terms of parameters of van Genuchten and Brooks-Corey functions using Eqs. (12) and (14), respectively to model the height of the capillary rise as:

$$Z = \frac{1}{\alpha_{BC} [1 + (q/K_s)]} + \frac{1}{1.3 n \alpha_{VG}} \left[\ln \left\{ 1 + \frac{K_s}{q} \right\} \right] \quad (15)$$

The van Genuchten parameters of α_{VG} and n were estimated by the water retention function of van Genuchten [20] as:

$$\theta = \theta_r + (\theta_s - \theta_r) [1 + (\alpha_{VG} h)^n]^{-m} \quad (16)$$

The parameters m should be equal to $1-1/n$ as a condition to arrive at a closed form of the theoretical hydraulic conductivity function of van Genuchten using the corresponding soil water retention function (Eq.16) in the relative hydraulic conductivity function of Mualem [19].

Now for developing the analytical model of the capillary rise based on the power-law hydraulic conductivity function, the hydraulic conductivity function proposed by Brooks-Corey [26] based on the corresponding soil water retention function and the relative hydraulic conductivity function of Burdine [27] was used and written as:

$$K(h) = K_s (\alpha_{BC} h)^{-\beta} \quad (17)$$

Where β is the dimensionless shape parameter. Embedding Eq. (17) in Eq. (3), the capillary rise profile as a function of the soil water suction head in the unsaturated soil above the water table is written as:

$$dz = \frac{dh}{1 + [\alpha_{BC}^\beta q / K_s] h^\beta} \quad (18)$$

The h value at the water table was taken as zero and at the ultimate height of the capillary rise front, the h value was taken as infinity as a limiting condition. Taking $t = 1 + [\alpha_{BC}^\beta q / K_s] h^\beta$ and corresponding limits of $t = 1$ at the water table and $t = \infty$ at the ultimate height of the capillary rise front, Eq. (18) is written as:

$$Z = \frac{1}{\beta [\alpha_{BC}^\beta q / K_s]^{1/\beta}} \int_1^\infty \frac{(t-1)^{(1/\beta)-1}}{t} dt \quad (19)$$

Taking $t_1 = t - 1$ with the limits of t_1 from 0 at the water table to infinity at the ultimate height of the capillary rise front, Eq. (19) is written as:

$$Z = \frac{1}{\beta \alpha_{BC} (q/K_s)^{1/\beta}} \int_0^\infty \frac{(t_1)^{(1/\beta)-1}}{t_1+1} dt_1 \quad (20)$$

The Eq. (20) can be written as:

$$Z = \frac{1}{\beta \alpha_{BC} (q/K_s)^{1/\beta}} \int_0^\infty \frac{(t_1)^{(1/\beta)-1}}{(t_1+1)^{(1-1/\beta)+1/\beta}} dt_1 \quad (21)$$

The Eq. (21) is of the form of the following complete Beta function as described by Andrews et al. [28].

$$B(p, q) = \int_0^\infty \frac{x^{q-1}}{(1+x)^{p+q}} dx = \frac{\pi}{\sin q\pi} \quad (22)$$

Therefore the Eq. (21) is written in the closed form as:

$$Z = \frac{1}{\beta \alpha_{BC} (q/K_s)^{1/\beta}} \left[\frac{\pi}{\sin \pi/\beta} \right] \quad (23)$$

The Eq. (23) is similar to the analytical model of capillary rise proposed by Cisler [6] and on rearranging it is written as:

$$Z = \left[\frac{\pi \operatorname{Cosec}(\pi/\beta)}{\beta \alpha_{BC}^\beta} \right] (K_s/q)^{1/\beta} \quad (24)$$

The parameter β in Eq. (24) is related to λ parameter of soil water function (Eq. 13) as $\beta = 2 + (l + 1)\lambda$ as given by Brooks-Corey [26]. The parameter l is the tortuosity factor which characterizes the combined effects of pore-connectivity and the flow path and is equal to 2.0 for the Burdine model of relative hydraulic conductivity.

PARAMETER ESTIMATION OF THE CAPILLARY RISE MODELS

The parameters n , α_{VG} , α_{BC} and $\beta (= 2 + (l + 1)\lambda)$ of these capillary rise models were estimated by RETC (RETention Curve) computer code as developed by van Genuchten et al. [29] by fitting the observed soil water retention data of Kalane et al. [30] in the Eqs. (13) and (16). While estimating these parameters, the other unknown parameters i.e. θ_r and θ_s as described in Eqs. (13) and (16) were also estimated. This code uses the weighted non-linear least-squares optimization approach based on the Marquardt-Levenberg's maximum neighborhood method given by Marquardt [31] such that the residual sum of squares (RSS) of the observed and the fitted soil water retention data $\theta(h)$ is minimized as the objective function $O(\mathbf{b})$:

$$O(\mathbf{b}) = \sum_{i=1}^N [w_i \{\theta_i - \hat{\theta}_i(\mathbf{b})\}]^2 \quad (25)$$

Where \mathbf{b} is the vector representing the unknowns, θ_i and $\hat{\theta}_i$ are the observed and the corresponding estimated soil water contents, respectively. N is the number of the soil water retention data points and equal to 9 in this analysis. The weighting factors w_i which reflect the reliability of the measured individual data were set equal to unity in this analysis

as the reliability of all the measured soil water retention data was considered equal. The fitting performance of the estimated soil water retention data with the observed data was evaluated with RSS and the coefficient of determination (r^2) which characterizes the relative magnitude of the total sum of squares associated with the fitted function as:

$$r^2 = \frac{\sum(\hat{\theta}_i - \bar{\theta}_i)^2}{\sum(\theta_i - \bar{\theta}_i)^2} \quad (26)$$

Where $\bar{\theta}_i$ is the mean of the observed soil water retention data. The values of the parameters of the capillary rise models n , α_{VG} , α_{BC} and β alongwith RSS and r^2 values are reported in Table 1.

PERFORMANCE EVALUATION OF CAPILLARY RISE MODELS

For evaluating these analytical models of the capillary rise, the estimated heights of the capillary rise (Table 2) were compared with the observed height of the capillary rise of 60 cm. from the water table for the corresponding capillary water fluxes in different soil textures i.e. sand, loamy sand, sandy loam, loam and silty clay loam using the observed saturated hydraulic conductivity data measured by Kalane et al. [30].

Table 1: Optimized values of parameters of soil water retention functions and fitting performance

Soil texture	Brooks-Corey function				van Genuchten function			
	Optimized values of parameters		Fitting performance		Optimized values of parameters		Fitting performance	
	λ	α_{BC} (cm ⁻¹)	RSS	r^2	n	α_{VG} (cm ⁻¹)	RSS	r^2
Sand	0.59 (3.77)*	0.106	2x10 ⁻⁵	0.999	1.85	0.071	2x10 ⁻⁵	0.999
Loamy sand	0.35 (3.05)	0.101	2x10 ⁻⁵	0.999	1.68	0.057	3x10 ⁻⁵	0.999
Sandy loam	0.28 (2.84)	0.090	5x10 ⁻⁵	0.999	1.69	0.046	9x10 ⁻⁵	0.998
Loam	0.42 (3.26)	0.045	9x10 ⁻⁵	0.997	2.38	0.025	18x10 ⁻⁵	0.995
Silty clay loam	0.21 (2.63)	0.032	9x10 ⁻⁵	0.996	2.41	0.018	10x10 ⁻⁵	0.996

* The data in parentheses are β values.

It is seen from Table 1 that the values of α_{BC} decreased as the fineness of the soil texture increased indicating height of the capillary fringe (inverse of α_{BC}) increased as the soils became comparatively more finer. The same trend was also observed for α_{VG} . The values of λ in general were observed to be less as the soils became finer indicating that the slope of the water retention function (curve) of the Brooks-Corey was observed to be more in light-textured soils in comparison with the medium-textured loam and moderately fine-textured silty clay loam soil. This showed that the porous medium of sand has comparatively less spread of pore-size distribution. Kosugi et al. [32] also reported that theoretically λ value approaches infinity for porous medium with uniform pore-size distribution whereas its value approaches a lower limit of zero for soils with wide range of pore sizes. It was also observed that the values of parameter n of van Genuchten function decreased as the sand content of these soils increased. The fitting behavior of the soil water retention functions is described by the combined effects of two parameters (α_{BC} , λ) in the Brooks-Corey function whereas in the van Genuchten function it is described by the three parameters (α_{VG} , n , m).

On perusal of the values of RSS and r^2 (Table 1) showed that in the coarse-textured soils of sand and loamy sand, both the soil water retention functions gave equal fitting performance while for the medium-textured soil of sandy loam and moderately fine-textured soils of loam and silty clay loam, the Brooks-Corey function gave comparatively better fit in comparison to the van Genuchten function. Mualem [33] also reported that there is no single soil water retention function that fits every soil. Mavimbela and Rensburg [34] also parameterized the soil water retention functions of Brooks-Corey and van Genuchten and reported that these functions fitted the measured soil water retention data with r^2 no less than 0.98.

Table 2: Estimated heights of capillary rise from the water table for the corresponding capillary water fluxes using analytical models

Soil texture	Observed capillary rise water flux q (cm/day) at capillary rise height of 60 cm*	Estimated heights of capillary rise using capillary rise models	
		Eq. (15)	Eq. (24)
Sand	0.32	51.6 (14.0)**	71.7 (19.5)
Loamy sand	0.25	49.8 (17.0)	60.7 (1.2)
Sandy loam	0.27	55.9 (6.8)	67.2 (12.0)
Loam	0.40	58.7 (2.2)	62.7 (4.5)
Silty clay loam	0.37	52.5 (12.5)	66.5 (10.8)

*Kalane et al. [30]

**Data in parentheses is the percent absolute deviation of the estimated heights from the observed height of the capillary rise of 60 cm.

On perusal of Table 2, It is seen that the capillary rise model based on the exponential function of the hydraulic conductivity proposed by Gardner alongwith using the parameter equivalence of conductivity functions of Gardner and Mualem-based van Genuchten underestimated the height of the capillary rise for all the soils under investigation while the capillary rise model based on power-law conductivity function of Brooks-Corey and in combination with the Burdine model of relative hydraulic conductivity overestimated the height of the capillary rise for the corresponding capillary water fluxes for these soils. The values of the heights of the capillary rise evaluated using these capillary rise models were found to be more closer (2.2 to 12.5 percent absolute deviations) to the observed values of the height of the capillary rise of 60 cm in the medium-textured porous media of sandy loam and loam and moderately fine-textured silty clay loam while more deviations (14.0 to 19.5 percent) were found between the observed and the estimated values of the capillary rise for the corresponding capillary water fluxes as the soil texture became comparatively more coarser.

CONCLUSIONS

The two analytical models of the capillary rise i.e. one based on the exponential hydraulic conductivity function of Gardner and its parameter correspondence with the Mualem-based van Genuchten hydraulic conductivity function and the other based on the power-law hydraulic conductivity function of Brooks-Corey based on the corresponding soil water retention function in combination with the Burdine model of relative hydraulic conductivity function were developed. These models based on the exponential and power-law conductivity functions underestimated and overestimated the capillary rise heights in comparison with the observed capillary rise height, respectively. The evaluated values of the capillary rise heights were found to be more closer to the observed values in the medium and moderately fine-textured soils.

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